MAY 2014

(Engineering Mathematics-I)

Roll No__________

Time: 3hrs BTAM - **101** **M.M : 60**

Note: Question-I is compulsory to attempt, consisting of ten short answer type questions carrying two marks each. Attempt five questions (carrying eight marks each) by selecting at least two questions each from Section -A and Section -B .

I(a) Identify the symmetries of the curve $r = \sin\frac{\theta}{2}$.

(b) If $u = F(\frac{x}{y}, \frac{y}{z}, \frac{z}{x})$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

(c) If $\vec{u}$ *and* $\vec{v}$ are irrotational. Vectors , then show that $\vec{u} \times \vec{v}$ is a solenoidal vector.

(d) Give the physical interpretation of curl of a vector point function

(e) Evaluate: $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x + y + z)\,dx\,dy\,dz$

(f) If $\vec{u}$ is a differentiable vector function of t of constant direction, then show that

$$\vec{u} \times \frac{du}{dt} = 0$$

(g) If $x = u(1+v)$ *and* $y = v(1+u)$, then find the value of $\frac{\partial(x,y)}{\partial(u,v)}$.

(h) Graph the set of points whose polar co-ordinates satisfy the inequalities

$$\pi/4 \le \theta \le 3\pi/4,\ 0 \le r \le 1$$

(i) Change the Cartesian integral $\int_{0}^{6}\int_{0}^{y} x\,dx\,dy$ into an equivalent polar integral .

(j) What surface is represented by $36x^2 + 9y^2 + 4z^2 = 36$?

Section-A

II(a)Trace the curve $y^2(a + x) = x^2(3a - x)$ by giving all salient features in detail.

(b)If ρ_1 and ρ_2 be radii of curvature at the ends of any chord of $r = 1 + \cos\theta$ which passes through the pole , then prove that

$$\rho_1^2 + \rho_2^2 = \frac{16}{9}$$

III (a) Show the volume of the solid generated by the revolution of the cycloid $x = a(\theta + \sin\theta), y = a(1 - \cos\theta), 0 \le \theta \le \pi$ about the tangent at vertex is $\pi^2 a^3$.

(b) Find the centre of gravity of the area between the curve $y^2(2a - x) = x^3$ and its asymptote.

IV (a) If $u = \sin^{-1}(x - y),\ x = 3t,\ and\ y = 4t^3$ then show that $\frac{du}{dt} = 3(1-t^2)^{-\frac{1}{2}}$

(b) State Euler's theorem and use it to prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u\log u,\quad whenever\ \ u = e^{x^2+y^2}$$

V(a) Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

(b) Expand $e^x \log(1+y)$ in ascending powers of x and y upto third degree terms.

SECTION-B

VI(a) Evaluate: $\int_0^\infty \int_0^x xe^{-\frac{x^2}{y}}\,dxdy$ by changing the order of integration.

(b) Find the volume of the tetrahydron bounded by the planes

$$x = 0, y = 0, z = 0, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \quad a, b, c \ge 0.$$

VII(a) If $r = \left|\vec{r}\right|$ and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{a}$ is a constant vector ,then prove that

$$\nabla \times \left(\frac{\vec{a} \times \vec{r}}{r^n}\right) = \frac{2-n}{r^n}\vec{a} + \frac{n(\vec{a}\cdot\vec{r})\vec{r}}{r^n}$$

(b) A vector field is given by $\vec{F} = \sin y\,\vec{i} + x(1+\cos y)\vec{j}$. Evaluate the line integral $\int \vec{F}\cdot\vec{dr}$ over a circular path given by $x^2 + y^2 = a^2, z = 0$.

VIII(a) Find the directional derivative of $f(x, y.z) = x\ y^2\ + yz^3$ at (2,-1,1) in the direction of normal to the surface $x\log z - y^2 = -4$ at (-1,2,1).

(b) Find the area lying outside the cardiode $r = a(1 - \cos\theta)$ and inside the circle $r = a\sin\theta$

IX(a) State Stoke's theorem and use it to evaluate $\int_C ydx + zdy + xdz$ where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$, *and the plane* $x + z = a$

(b) Verify Green's theorem for $\oint_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$, where C is the Boundary of the region bounded by the lines $x = 0, y = 0, and\ x + y = 1$

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